

DFTT 71/92
December 1992

THE KAZAKOV-MIGDAL MODEL AS A HIGH TEMPERATURE LATTICE GAUGE THEORY

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Abstract

We show that the Kazakov-Migdal (K-M) induced gauge model in d dimensions describes the high temperature limit of ordinary lattice gauge theories in $d + 1$ dimensions. The matter fields are related to the Polyakov loops, while the spatial gauge variables become the gauge fields of the K-M model. This interpretation of the K-M model is in agreement with some recent results in high temperature lattice QCD.

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1. Introduction

Recently V.Kazakov and A.Migdal proposed a new lattice gauge model, in which the gauge self-interaction is induced by scalar fields in the adjoint representation [1]. The action they propose is defined on a generic d -dimensional lattice and has the following form:

$$S = \sum_x N \text{Tr} [m^2 \phi^2(x) - \sum_\mu \phi(x) U(x, x + \mu) \phi(x + \mu) U^\dagger(x, x + \mu)] \quad (1)$$

where $\phi(x)$ is an Hermitian $N \times N$ matrix defined on the sites x of the lattice and $U(x, x + \mu)$ is a unitary $N \times N$ matrix, defined on the links $(x, x + \mu)$, and plays the role, as in the usual lattice discretization of Yang-Mills theories, of the gauge field. Integrating over the scalar field ϕ one can induce an effective action for the gauge field,

$$\int DUD\Phi \exp(-S) \sim \int DU \exp(-S_{ind}[U]) \quad (2)$$

with:

$$S_{ind}[U] = -\frac{1}{2} \sum_\Gamma \frac{|\text{Tr} U[\Gamma]|^2}{l[\Gamma] (2m^2)^{l[\Gamma]}} \quad , \quad (3)$$

where $l[\Gamma]$ is the length of the loop Γ , $U[\Gamma]$ is the ordered product of link matrices along Γ and the summation is over all closed loops.

The model turns out to be solvable in the large N limit, for any space-time dimension d [1, 2, 3] and exactly solvable, for any value of N in $d=1$ [4]. A further interesting feature of the model is its deep connection with the theory of non-critical strings, in particular it can be shown that in the $d = 1$ case the K-M model is equivalent to the vortex free sector of the $d=1$ string [5]. Despite all these nice features the model seems to miss the original goal of Kazakov and Migdal, since the induced gauge theory (3), due to the fact that the matter fields are in the adjoint representation, has a super-confining behaviour [6] and seems to have the wrong perturbative vacuum [7, 8]. Several improvement of the original K-M action have been proposed to avoid these problems [8, 9, 10, 11], but even if many interesting results have been obtained the problem of the identification with a pure lattice gauge theory of the ordinary type (namely with the ordinary confinement behaviour) while keeping exact solvability is still open.

In this letter we suggest a new point of view on the problem and show that the K-M model can be identified *before integrating over the matter fields* with an ordinary $SU(N)$ lattice gauge theory in the high temperature limit, the matter fields being related to the Polyakov loops, while the spatial gauge variables become the gauge fields of the K-M model.

This letter is organized as follows: after a short introduction (just to set notations) on finite temperature lattice gauge theory (sect.2), we describe and discuss the equivalence with the K-M action in sect.3 and 4. Sect.5 is devoted to the continuum limit and sect.6 to some concluding remarks.

2. Finite temperature lattice gauge theories

Let us consider a pure gauge theory with gauge group $SU(N)$ defined on a $d+1$ dimensional cubic lattice. In order to describe a finite temperature LGT we must take periodic boundary conditions in one direction (which we shall call from now on “time-like” direction), while the boundary conditions in the other d direction (which we shall call “space-like”) can be chosen freely. Let us take a lattice of N_t (N_s) spacings in the time (space) direction. The theory will contain only gauge fields described by the link variables $U_{n;i} \in SU(N)$ where $n \equiv (\vec{x}, t)$ denotes the space-time position of the link and i its direction. It is useful to choose different couplings in the time and space directions. Let us call them β_t and β_s respectively. Let us take the simplest choice for the lattice gauge action, namely the Wilson action:

$$S_W = \sum_n N \operatorname{Re} \left\{ \beta_t \sum_i \operatorname{tr}(U_{n;0i}) + \beta_s \sum_{i < j} \operatorname{tr}(U_{n;ij}) \right\} \quad (4)$$

where $U_{n;0i}$ ($U_{n;ij}$) are the time-like (space-like) plaquette variables, defined as usual:

$$U_{n;ij} = U_{n;i} U_{n+\hat{i};j} U_{n+\hat{j};i}^\dagger U_{n;j}^\dagger \quad (5)$$

β_s and β_t are related to the (bare) coupling constant g and the temperature T of the gauge theory by the relations ¹:

$$\frac{2}{g^2} = a^{3-d} \sqrt{\beta_s \beta_t}, \quad T = \frac{1}{N_t a} \sqrt{\frac{\beta_t}{\beta_s}} \quad (6)$$

where a is the space-like lattice spacing while $\frac{1}{N_t T}$ is the time-like spacing. The two are related by the adimensional ratio $\epsilon \equiv \frac{1}{N_t T a}$. We can solve the above equations in terms of ϵ as follows:

$$\beta_t = \frac{2}{g^2 \epsilon} a^{d-3} \quad (7)$$

$$\beta_s = \frac{2\epsilon}{g^2} a^{d-3} \quad (8)$$

In a finite temperature discretization it is possible to define gauge invariant variables which are topologically non-trivial loops, closed due to the periodic boundary conditions in the time directions. The simplest choice is the Polyakov loop defined as follows:

$$P(\vec{x}) = \operatorname{tr} \prod_{t=1}^{N_t} (U_{\vec{x},t;0}) \quad (9)$$

¹Notice that the coupling constant g is rescaled here by a factor N with respect for instance to the notations of ref. [12]

where \vec{x} labels the space coordinates of the lattice sites. Moreover, an important feature of the finite temperature theory with respect to the zero temperature case is that it has a new global symmetry (independent from the gauge symmetry) with symmetry group the center C of the gauge group (in our case Z_N). The Polyakov loop turns out to be a natural order parameter for this symmetry.

It is possible to obtain, just from the definition itself of the model, some general properties in the high temperature regime (see for instance [12]). In this region the symmetry with respect of the center of the group is broken, the theory is deconfined, the Polyakov loop has a non-zero expectation value and, what is more important, it is an element of the center of the gauge group. Physically this means that the links in the time direction fluctuate around one of the N^{th} roots of unity of the Z_N group. We can lift this degeneracy by adding to the Wilson action a “magnetic term”

$$S_m = N h \sum_{\vec{x}} Re \ tr \prod_{t=1}^{N_t} (U_{\vec{x},t;0}) \quad (10)$$

and eventually send $h \rightarrow 0$. This procedure selects the vacuum in which $\langle P(\vec{x}) \rangle = 1$.

The spatial links are static up to gauge transformations and this fact tells us that the spatial degrees of freedom of the model behave as if they belonged to a zero temperature model in one dimension less and with coupling constant $g^2 T$. If one could integrate out these spatial degrees of freedom the resulting effective action for the Polyakov loops would be that of a d -dimensional spin model with short range spin-spin interaction [12].

3. The K-M model as an high temperature LGT.

Let us take the $d + 1$ dimensional LGT described above (with gauge group $SU(N)$) and consider first a very special case, namely $N_t = 1$. We will be interested in the $T \rightarrow \infty$ (hence $\epsilon \rightarrow 0$) limit.

Let us call S_t (S_s) the contribution of the temporal (spatial) plaquettes to the action, with a “magnetic term” of the type (10) included in S_t . Due to the boundary conditions and to the $N_t = 1$ position we can rewrite S_t as:

$$S_t = N \beta_t \sum_{\vec{x},i} Re \ tr \left\{ U_{\vec{x};i} U_{\vec{x}+\hat{i};0} U_{\vec{x};i}^\dagger U_{\vec{x};0}^\dagger \right\} + N h \sum_{\vec{x}} Re (U_{\vec{x};0}) \quad (11)$$

This model is interesting in itself and represents the generalization of the K-M model to “matter” fields described by unitary matrices. Notice also that since the space contribution S_s to the action is depressed as $\beta_s \sim 1/T$, in the $T \rightarrow \infty$ limit this model should capture most of the features of high temperature LGT.

Following Svetitsky and Yaffe [12] one can argue at this point that in the high temperature limit each temporal plaquette fluctuates around the identity . It is easy to see that the fluctuations are of order $1/\sqrt{\beta_t}$. In fact if we put $U_{pl} = e^{i\frac{\phi}{\sqrt{\beta_t}}}$ with ϕ a traceless hermitian matrix we have:

$$\int dU_{pl} e^{N\beta_t \text{tr} \text{Re} U_{pl}} = \frac{e^{N^2\beta_t}}{\sqrt{\beta_t}} \int d\phi e^{-N \text{tr} \phi^2} [1 + O(1/\beta_t)] \quad (12)$$

Correspondingly the link variables in the time direction fluctuate around one of the N^{th} roots of unity of the Z_N group with fluctuations still of order $1/\sqrt{\beta_t}$. When the magnetic term is switched on the vacuum is forced to be $U_{\vec{x};0} = \mathbf{1}$ and we can expand $U_{\vec{x};0}$ in the following way:

$$U_{\vec{x};0} \equiv e^{i\frac{\phi(\vec{x})}{\sqrt{\beta_t}}} = \mathbf{1} + i\frac{\phi(\vec{x})}{\sqrt{\beta_t}} - \frac{\phi^2(\vec{x})}{2\beta_t} + \dots \quad (13)$$

By inserting (13) in (11) we find:

$$S_t = N \beta_t \text{tr} \left\{ d \Omega_x \mathbf{1} + \frac{1}{\beta_t} \sum_{\vec{x}} \left(-m^2 \phi(\vec{x})^2 + \sum_{i=1}^d U_{\vec{x};i} \phi(\vec{x}) U_{\vec{x};i}^\dagger \phi(\vec{x} + \hat{i}) \right) \right\} \quad (14)$$

where $m^2 = d + \frac{h}{2\beta_t}$ and Ω_x is the volume of the d dimensional space. In ref. [12] only the constant leading contribution of order β_t was considered; here we recognize that the contributions of the fluctuations, of order 1, coincide, apart from an irrelevant constant, with the K-M action with arbitrary m^2 (eq. (1)) and in the limit of zero magnetic field with the K-M action at the critical point $m^2 = d$. The expansion (13) and in particular the choice of the expansion parameter $\frac{1}{\sqrt{\beta_t}}$ is also justified *a posteriori* by the analysis of the distribution of the eigenvalues of $\phi(\vec{x})$ resulting from the action (14). It was shown in [3] that at least in the large N limit and for $m^2 > d$ the eigenvalue distribution of $\phi(\vec{x})$ is semicircular with a finite, non-zero radius. This implies that the eigenvalues of the original unitary matrix $U_{\vec{x};0}$ are restricted to a region of order $\frac{1}{\sqrt{\beta_t}}$ around 1 on the unit circle, thus justifying the expansion (13). For $d > 1$ such distribution of the eigenvalues survives also in the limit $h \rightarrow 0$ (with $h > 0$) namely $m^2 \rightarrow d$, whereas for $d = 1$ the radius of the eigenvalue distribution goes to infinity as $h \rightarrow 0$ ($m^2 \rightarrow 1$) . This shows that in the one dimensional case the critical point can only be described in the unitary matrix formulation given by (11).

Let us consider now the case of an arbitrary N_t . The action (without the contribution of spatial plaquettes) is:

$$S_t = N\beta_t \sum_{\vec{x},i} \sum_{t=1}^{N_t} \text{Re} \text{tr} \{ U_{\vec{x},t;i} U_{\vec{x}+\hat{i},t;0} U_{\vec{x},t+1;i}^\dagger U_{\vec{x},t;0}^\dagger \} + N h \sum_{\vec{x}} \text{Re} P(\vec{x}) \quad (15)$$

where $P(\vec{x})$ is the Polyakov loop given in (9). It is possible to choose the gauge in such a way that $U_{\vec{x},t;0} = 1$ for $t = 1, 2, \dots, N_t - 1$ so that the only non trivial links in the time direction are $U_{\vec{x},N_t;0}$, related to the Polyakov loop by the relation $P(\vec{x}) = \text{tr} U_{\vec{x},N_t;0}$. The plaquettes at $t < N_t$ then reduce to $U_{\vec{x},t;i} U_{\vec{x},t+1;i}^\dagger$. It follows that in the large β_t limit the spatial link variables at different values of t coincides up to fluctuations of order $1/\sqrt{\beta_t}$. So we can put

$$U_{\vec{x},t;i} = U_{\vec{x};i} \exp\left\{i \frac{\psi_i(\vec{x}, t)}{\sqrt{\beta_t}}\right\} \quad (16)$$

where we are free to choose $\psi_i(\vec{x}, 1) = 0$. In complete analogy with the case $N_t = 1$ we also have:

$$U_{\vec{x},N_t;0} = \exp\left\{i \phi(\vec{x}) \sqrt{\frac{N_t}{\beta_t}}\right\} \quad (17)$$

where the factor $\sqrt{N_t}$ in the exponent at the r.h.s. of (17) is needed, as we shall see, for a smooth $N_t \rightarrow \infty$ limit. By inserting equations (16) and (17) into the action (15) and by expanding in powers of $1/\beta_t$ up to terms of order 1, one obtains an action quadratic in the $\psi_i(\vec{x}, y)$'s:

$$\begin{aligned} S_t = & N \text{tr} \sum_{\vec{x};i} \left\{ N_t \beta_t + \sum_{t=2}^{N_t-1} [\psi_i(\vec{x}, t) \psi_i(\vec{x}, t+1) - \psi_i^2(\vec{x}, t)] - \psi_i^2(\vec{x}, N_t) - \right. \\ & - \sqrt{N_t} \psi_i(\vec{x}, N_t) [\phi(\vec{x} + \hat{i}) - U_{\vec{x};i}^\dagger \phi(\vec{x}) U_{\vec{x};i}] - \frac{N_t}{2} [\phi(\vec{x} + \hat{i}) - \\ & \left. - U_{\vec{x};i}^\dagger \phi(\vec{x}) U_{\vec{x};i}]^2 - N h \frac{N_t}{\beta_t} \phi^2(\vec{x}) \right\} \end{aligned} \quad (18)$$

The fluctuations $\psi_i(\vec{x}, t)$ can be eliminated by performing the corresponding gaussian integrals and the final result coincides with the K-M model as given by eq.(14)². The same result can be obtained also by noticing that, as the self interaction term is neglected, each spatial link variable $U_{\vec{x},t;i}$ belongs to just two temporal plaquettes and that the corresponding integral can be done by using character expansion in analogy to lattice QCD in two dimensions. This allows to define a “renormalized” coupling β_{eff} through the relation:

$$I_f(\beta_{eff}) = \left[\frac{I_f(\beta)}{I_0(\beta)} \right]^{N_t} \quad (19)$$

where I_f and I_0 are the coefficients of the fundamental and identity characters in the character expansion of the action. Then it can be shown that the large β behaviour of $\beta_{eff}(\beta)$ is as expected $\beta_{eff} \sim \frac{\beta}{N_t}$ (see for instance ref. [13]).

The final result eq.(14) is then independent from the number N_t of lattice spacing in the time direction provided the field $\phi(\vec{x})$ is normalized as in (17). Notice that

²Notice that Ω_x in (14) is now replaced by the volume of the whole space-time lattice

the r.h.s. of (17) is also independent of N_t according to the definition of β_t given in (7). So the continuum limit in the time dimension $N_t \rightarrow \infty$ is well defined and leads to the K-M model.

Hence we can conclude that

for $d > 1$ the d -dimensional K-M model is a good description of the small fluctuations of the Polyakov loops around their minimum (frozen) position in the high temperature limit of a pure lattice gauge theory in $d + 1$ dimensions with the Wilson action.

4. Comments and remarks

The integration over the ϕ fields in (14) leads to the induced gauge action eq.(3) which can now be interpreted as describing the spatial degrees of freedom of a pure LGT at $T = \infty$. Since, as mentioned above, the space degrees of freedom in the high temperature limit behave as a zero temperature d -dimensional gauge theory with coupling constant $g_{eff}^2 = g^2 T$, the K-M model corresponds to the $g_{eff} = \infty$ point of the theory thus leading to the superconfinement behaviour.

At $T < \infty$ one has to add a small (order $1/T$) gauge self-interaction term. This term obviously destroys the exact solvability of the model, but it can be treated as a small perturbation around the K-M solution. The main outcome of our analysis is probably in the fact that we now understand that such a perturbative analysis would be an high-temperature expansion for the LGT.

Let us consider for instance the expectation value of a non backtracking Wilson loop which is zero at $T = \infty$ due to the superconfinement. Its first non trivial perturbative contribution in the $1/T$ expansion is obtained by filling the loop by elementary plaquettes³. The resulting “filled Wilson loops” is invariant under the local Z_N symmetry and has to be evaluated using the measure of the pure M-K induced action. This was done in ref. [7] where it was shown that the “filled Wilson loop” has an area law behaviour. It is also remarkable and not quite understood yet that in this context intriguing connections with 2d spin models emerge. Our interpretation is that the “filled Wilson loop” is the first non vanishing term in a large temperature expansion of the vacuum expectation value of the ordinary Wilson loop.

The K-M model is *only* apt to describe fluctuations around one given vacuum of the high temperature LGT; in other words it can only describe finite temperature LGT in the broken Z_N phase. However, even in the framework of the K-M model

³In more than two spatial dimension one has to sum over the contributions of all surfaces made of elementary plaquettes and having the original loop as a boundary. These contributions are of order $T^{-\mathcal{A}}$ where \mathcal{A} is the area of the surface, so that the surface of minimal area dominates at high temperature.

one has at least a qualitative understanding of the restoring of the Z_N symmetry below the critical temperature. In fact the expectation value of the Polyakov loop

$$\langle P(\vec{x}) \rangle = \langle \text{Tr} e^{i\phi(\vec{x}) \sqrt{\frac{N_t}{\beta_t}}} \rangle \quad (20)$$

can be evaluated in the large N limit by replacing $\phi(\vec{x})$ with its classical value given in ref. [3]. At the critical point $m^2 = d$ this is given by a semicircular distribution of eigenvalues of radius $r = \sqrt{\frac{2d-1}{d(d-1)}}$. At high temperature, more precisely for $r\sqrt{\frac{N_t}{\beta_t}} \ll \pi$, the eigenvalues are peaked around one vacuum and the Z_N symmetry is broken. However for values of β_t such that $r\sqrt{\frac{N_t}{\beta_t}} \approx \pi$ the eigenvalues of $\sqrt{\frac{N_t}{\beta_t}}\phi(\vec{x})$ are distributed over several vacua on most of the unit circle and so the symmetry is eventually restored.

Without a more detailed understanding of the symmetry restoration nothing new can be said in this context on the Svetitsky-Yaffe conjecture that the critical behavior of a $(d+1)$ -dimensional finite temperature gauge theory is the same as that of a d -dimensional spin model with the same Z_N symmetry. At this stage at least the emergence of spin models [7] in the expectation value of the Wilson loop does not appear to be correlated with the Svetitsky-Yaffe conjecture. It should be noticed however that if the contribution of the spatial plaquettes is neglected, an effective theory for the Polyakov loop valid also in the region of large $\phi(\vec{x})$ could in principle be obtained directly from (15) by integrating over the spatial links variables. This would require the use of a generalization of the Itzykson-Zuber formula described in [7] which is exact for $N = 2$ and valid as an asymptotic formula for $N > 2$.

The instability which occurs in the $m^2 < d$ region, corresponding to negative values of h in (15), can be understood from the high temperature LGT point of view as a consequence of the fact that with our “magnetic field” in this regime we are actually pushing the system *out* of the chosen vacuum which becomes unstable. Similarly the instability due to the presence of a linear term [14] which occurs if one looks at $U(N)$ instead of $SU(N)$ K-M models, can be understood as a signature of the Goldstone modes of the broken $U(1)$ symmetry which one has in this case.

Let us finally consider the case $d = 1$. Spatial plaquettes are absent in this case and eq. (15) gives the complete action irrespective of the value of β_t . This is just two dimensional QCD on a cylinder or, in case the spatial dimension is also compactified, on a torus. We know from ref. [3] that in the $d = 1$ case at the critical point $m^2 = 1$ the distribution of the eigenvalues of $U_{\vec{n}, N_t; 0}$ is not confined to a small region of the unit circle. As a consequence we are always in the unbroken phase and the description of the system in terms of the K-M model is not valid. On the other hand by expanding both spatial and temporal links in a fashion similar to eq.(16) one can reduce the action of eq. (15) with periodic boundary conditions in both space and time to an action containing only one space and one time-like link, in agreement with general result on lattice QCD in two dimensions [11].

5. Taking the continuum limit

In the previous sections we have derived the K-M model as a high temperature limit of lattice QCD. However while in ordinary lattice QCD one has a well defined continuum limit, this is not the situation in the K-M model with a quadratic potential at the critical point $m^2 = d$. It was pointed out in [3] that if one approaches $m^2 = d$ from below there exists a solution of the master field equation corresponding to a semicircular distribution of eigenvalues whose radius r goes to infinity as $m^2 \rightarrow d$, a signal of the existence of a critical point. Such configuration however is a local maximum of the free energy; moreover the free energy itself is unbounded from below for $m^2 < d$. This can be cured by adding a higher order term to the action, for instance a quartic term $\frac{\lambda}{4} \text{Tr} \phi^4$. From the point of view of the high temperature expansion this is not such an *ad hoc* adjustment as it might appear at first, in fact the continuum limit corresponds to the limit where the radius of the eigenvalue distribution goes to infinity and in that limit higher order terms in $\phi(\vec{x})$ are expected to be relevant. Therefore in the continuum limit the high temperature QCD is most likely described by a K-M model with a higher order potential in agreement, as discussed below, with some recent results on lattice QCD [16, 17]. It has indeed been shown in [15] that, with the addition of a quartic term $\frac{\lambda}{4} \text{Tr} \phi^4$ to the action, a second order critical point can be reached, at least in the case of SU(2) in $d=4$. The analysis of the phase diagram can be done rather easily within a mean field approximation: in the λ, m^2 plane one finds a line of first order transitions starting from the K-M critical point and ending with a second order critical point located at $\lambda = 2.57$ and $m^2 = 2.26^4$. Moreover direct Montecarlo simulations show that these mean field results are quite accurate and even the critical point location is essentially confirmed by the computer simulation. We have made a similar mean field analysis in the $d = 3$ case, in which we are interested, showing a similar scenario: a line of first order phase transitions starting from $m^2 = 3$ and ending with a second order critical point located at $\lambda \sim 2.2, m^2 \sim 1.56$. In [15] it was stressed that the corresponding continuum theory had nothing to do with ordinary SU(2) gauge theory. Our suggestion is that *it should instead describe the high temperature, deconfined phase of SU(2)* (with one more space-time dimension). This conjecture is in remarkable agreement with some recent results on the high temperature behaviour of lattice gauge theories, both within a perturbative approach [16], and with montecarlo simulations [17]. The main idea behind [16, 17] is that (3+1) dimensional gauge theories at high temperature undergo a peculiar form of dimensional reduction. The original picture [18] of a complete dimensional reduction (namely a complete decoupling of the degrees of freedom in the compactified time direction, which would lead to an effective three-dimensional theory of the ordinary type) does not occur in general and one finds instead a three dimensional gauge theory coupled with a scalar field ϕ in the adjoint representation with a non

⁴Notice that there is a factor of two between our normalization of the mass parameter and that of [15]

trivial potential $V(\phi)$. This potential has a quadratic term with the wrong sign to be identified as a mass term and a quartic contribution which stabilizes the action, exactly as in our mean field solution. Moreover the action used in [17] (eq.(20) of [17]) for the montecarlo simulation is exactly that of a K-M model in the instable ($m^2 < 3$) region, with a quartic term (like the above described mean field solution) plus the gauge self-interaction term. Taking into account the obvious uncertainties of our mean field estimate the numerical agreement on the coefficient of the quartic contribution between our result and that of [17] is impressive.

6. Conclusions

In this letter we have shown that, besides the usual identification with a LGT coupled with adjoint matter in the strong coupling regime, the Kazakov Migdal model can also be related to the behaviour of a pure lattice gauge theory (with Wilson action) in the limit of very high temperature. In particular we have seen that the ϕ fields describe the small fluctuations around the frozen position of the Polyakov loops.

We think that this new point of view is important if we want to understand some of the peculiar features of the K-M model (superconfinement, lack of ordinary perturbative vacuum, presence of an unstable phase for $m^2 < d$ and of a finite gap in the free energy), but besides this we think that there are two other reasons of interest. First the master field solution of the K-M model could give us a powerful tool to do both analytical and numerical calculations in pure LGT at high temperature. Second, more ambitious point: the K-M model is related to non critical strings, although this relation is still not quite clear for $d > 1$ (see however [19] and [20] for some recent progress on this subject). Our interpretation might lead (at least in the high temperature, large N limit) to a first description in terms of strings of LGT, a hope which justifies by itself further efforts toward a better understanding of the K-M model.

Note added

While completing this letter we received a new interesting paper by Dobroliubov, Kogan, Semenoff and Weiss [21], in which the interpretation of the “filled Wilson loop” as the result of the perturbative expansion of gauge self-interaction term is discussed. The two regimes discussed in [21] may be identified within our approach as the high temperature deconfined regime and the low temperature confined phase

respectively, and the coupling λ of [21] with our ratio $\frac{\beta_s}{\beta_t}$ ⁵.

Acknowledgments

One of us (A.D.) thanks J. Ambjørn and D. Boulatov for useful discussion and in particular J. Ambjørn for pointing out to us the relevance of ref. [16] and [17].

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⁵Notice, in order to avoid confusion that our high temperature phase is what is called in [21] the low temperature one and is characterized by a K-M behaviour and by $\lambda \equiv \frac{\beta_s}{\beta_t} \rightarrow 0$

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